# STRUCTURE OF M.Sc. – MATHEMATICS SYLLABUS

| I-Semester | | | | | | | |
|---|---|---|---|---|---|---|---|
| Subject | Papers | Instruction Hrs/week | Duration of exam (hr) | Marks | | | Credits |
| | | | | IA | Exam | Total | |
| Theory | Algebra-I | 4 | 3 | 30 | 70 | 100 | 4 |
| | Real Analysis | 4 | 3 | 30 | 70 | 100 | 4 |
| | Topology-I | 4 | 3 | 30 | 70 | 100 | 4 |
| | Ordinary differential equation | 4 | 3 | 30 | 70 | 100 | 4 |
| | Discrete Mathematics | 4 | 3 | 30 | 70 | 100 | 4 |
| Practicals | I. Scilab& Maxima practicals& problem working | 4 | 3 | 15 | 35 | 50 | 2 |
| | II. Discrete Mathematics practicals using Maxima or such other software & problem working | 4 | 3 | 15 | 35 | 50 | 2 |
| Total of Credits per semester | | | | | | | 24 |

| II-Semester | | | | | | | |
|---|---|---|---|---|---|---|---|
| Subject | Papers | Instruction Hrs/week | Duration of exam (hr) | Marks | | | Credits |
| | | | | IA | Exam | Total | |
| Theory | Algebra-II | 4 | 3 | 30 | 70 | 100 | 4 |
| | Complex Analysis | 4 | 3 | 30 | 70 | 100 | 4 |
| | Topology-II | 4 | 3 | 30 | 70 | 100 | 4 |
| | Partial differential equation | 4 | 3 | 30 | 70 | 100 | 4 |
| | Functional Analysis | 4 | 3 | 30 | 70 | 100 | 4 |
| Practicals | I.Partial differential equation practicals& problem working | 4 | 3 | 15 | 35 | 50 | 2 |
| | II.Mathematical Modelling and Numerical Analysis -I | 4 | 3 | 15 | 35 | 50 | 2 |
| Total of Credits per semester | | | | | | | 24 |

| III-Semester | | | | | | | |
|---|---|---|---|---|---|---|---|
| Subject | Papers | Instruction Hrs/week | Duration of exam (hr) | Marks | | | Credits |
| | | | | IA | Exam | Total | |
| Theory | Differential Geometry | 4 | 3 | 30 | 70 | 100 | 4 |
| | Fluid Mechanics | 4 | 3 | 30 | 70 | 100 | 4 |
| | Computational linear Algebra | 4 | 3 | 30 | 70 | 100 | 4 |
| Departure elective | A. Mathematical Methods<br>B. Numerical Analysis | 4 | 3 | 30 | 70 | 100 | 4 |
| Open elective | Decision theory | 4 | 3 | 30 | 70 | 100 | 4 |
| Practicals | I. Computational linear Algebra | 4 | 3 | 15 | 35 | 50 | 2 |
| | II. Mathematical Methods and Numerical Analysis | 4 | 3 | 15 | 35 | 50 | 2 |
| Total of Credits per semester | | | | | | | 24 |

| IV Semester | | | | | | | |
|---|---|---|---|---|---|---|---|
| Subject | Papers | Instruction Hrs/week | Duration of exam (hr) | Marks | | | Credits |
| | | | | IA | Exam | Total | |
| Theory | Measure and Integration | 4 | 3 | 30 | 70 | 100 | 4 |
| Elective | A.Riemannian Geometry<br>B. Special Functions<br>C. Theory of numbers<br>D. Enter And Meromorphic Function | 4 | 3 | 30 | 70 | 100 | 4 |
| Elective | A.Magnetohydrodynamics<br>B. Fluid Dynamics Of Ocean And Atmosphere<br>C. Computational Fluid Dynamics(CFD) | 4 | 3 | 30 | 70 | 100 | 4 |
| Elective | A.Finite Element Method With Applications<br>B. Graph Theory<br>C. Design And Analysis of Algorithm | 4 | 3 | 30 | 70 | 100 | 4 |
| Project | | 4 | 3 | 30 | 70 | 100 | 4 |
| Total of Credits per semester | | | | | | | 20 |
| Program Grand Total of Credits | | | | | | | 92 |

**M.Sc.-Mathematics**
**FIRST SEMESTER**

## <u>ALGEBRA-I</u>
**(4 Hours/Week)**

Recapitulation: Groups, Subgroups, Cyclic groups, Normal Subgroups, Quotient groups, Homomorphism, Types of homomorphism. **(1 hour)**

Permutation groups, symmetric groups, cycles and alternating groups, dihedral groups, Isomorphism theorems and its related problems, Automorphism, Inner automorphism, groups of automorphisms and inner automorphisms and their relation with centre of a group. **(8 hours)**

Group action on a set, Orbits and Stabilizers, The orbit-stabilizer theorem, The Cauchy-Frobenius lemma, Conjugacy, Normalizers, Centralizers, Class equation of a finite group and its applications. **(7 hours)**

Sylow's groups and subgroups, Sylow's theorems for a finite group, Applications and examples of p-Sylow subgroup. **(7 hours)**

Solvable groups, Simple groups, Applications and examples of solvable and simple groups, Jordan-Holder Theorem. **(7 hours)**

Recapitulation: Rings, Some special classes of rings (Integral domain, Division ring, field). **(1 hours)**

Homomorphism of rings, Kernel and image of Homomorphism of rings, Isomorphism of rings, Ideals and Quotient rings, Fundamental theorem of Homomorphism of rings. **(8 hours)**

Theorems on principle, maximal, and prime ideals, Field of quotients of an integral domain, Imbedding of rings. **(7 hours)**

Euclidian rings, Prime and relatively prime elements of a Euclidian ring, Unique factorisation theorem, Fermat's theorem, Polynomial rings, the division Algorithm. **(7 hours)**

Polynomials over the rational field, Primitive polynomial, Content of a polynomial, Gauss lemma, Eisenstein criteria, Polynomial rings over Commutative rings, Unique Factorization Domains. **(7 hours)**

## TEXT BOOKS

1. I. N. Herstein : Topics in Algebra, 2nd Edition, Vikas Publishing House, 1976.
2. Surjeet Singh and Qazi Zameeruddin, Modern Algebra, Vikas Publishing House,1994.
3. N. Jacobson : Basic Algebra-I, HPC, 1984.

## REFERENCE BOOKS

1. M. Artin : Algebra, Prentice Hall of India, 1991.
2. Darek F. Holt, Bettina Eick and Eamonaa. Obrien. Handbook of Computational groupTheory, Chapman and Hall, CRC Press, 2005.
3. J. B. Fraleigh : A First Course in Algebra, $7^{th}$Edition, Addison-Wesley Longman,2002.

## REAL ANALYSIS

**(4 Hours/Week)**

The Riemann-Stieltjes Integral : Definition and existence of the integral, Linear properties of the integral, the integral as the limit of sums, Integration and Differentiation, Integration of vector valued function . Functions of bounded variation - First and Second Mean Value Theorems, Change of variable rectifiable curves. **(21 hours)**

Sequence and series of function : Pointwise and Uniform Convergence, Cauchy Criterion for uniform convergence, Weierstrass M-Test, Uniform convergence and continuity, Uniform Convergence and Riemann-Stieltjes Integration, Uniform convergence and Differentiation, Uniform convergence and bounded variation-Equacontinuous families functions, Uniform convergence and boundedness, The Stone-Weierstrass theorem and Weierstrass approximation of continuous function, illustration of theorem with examples - properties of power series, exponential and logarithmic functions, trigonometric functions. Topology of $R^n$, K-cell and its compactness, Hein-Borel Theorem. Bozano-Weierstrass theorem, Continuity, Compactness and uniform continuity. **(21 hours)**

Functions of several variables, Continuity and Differentiation of vector-valued functions, Linear transformation of Rk properties and invertibility, Directional Derivative, Chain rule, Partial Derivative, Hessain matrix. The Inverse Functions Theorem and its illustrations with examples. The Implicit Functions Theorem and illustrations and examples. The Rank theorem illustration and examples. **(18 hours)**

## TEXT BOOKS

1. W. Rudin : Principles of Mathematical Analysis, McGraw Hill, 1983.
2. T. M. Apostol : Mathematical Analysis, New Delhi, Narosa, 2004.

## REFERENCE BOOKS

1. S. Goldberg : Methods of Real Analysis, Oxford and IBH, 1970.
2. J. Dieudonne : Treatise on Analysis, Vol. I, Academic Press, 1960.

## TOPOLOGY - I

**(4 Hours/Week)**

Finite and Infinite sets, Denumerable and Non-denumerable sets, Countable and Uncountable sets, Equivalent sets, Concept of Cardinal numbers, Schroeder-Bernstein theorem, Cardinal number of a power sets - Addition of Cardinal numbers, Exponential of Cardinal numbers, Examples of Cardinal Arithmetic, Cantor's Theorem, Card X<Card P(X), Relations connecting $K_0$ and c, Continuum Hypothesis, Zorn's lemma . **(16 hours)**

Definition of a metric, Bolzano-Weierstrass theorem, Open and closed balls. Cauchy and convergent sequences, Complete metric spaces. Continuity, Contraction mapping theorem, Banach fixed point theoremBounded and totally bounded set, Cantor's Intersection theorem. Nowhere dense sets, Baire's category theorem, Isometry, Embedding of a metric space in a complete metric space. **(14 hours)**

Topology : Definition and examples, Open and Closed sets, Neighbourhoods and Limits points, Closure, Interior and Boundary of a set. Relative Topology, Bases and Sub-bases, Continuity and Homeomorphism, Pasting lemma. **(16 hours)**

Connected spaces : Definition and examples, Connected sets in the real line, Intermediate value theorem, Components and path components, local connectedness and path connectedness. **(14 hours)**

**TEXT BOOKS**

1. J. R. Munkres : Topology - A First Course - Prentice Hall of India, 1996.
2. W. J. Pervin : Foundations of General Topology - Academic Press, 1964.

**REFERENCE BOOKS**

1. G. F. Simmons : Introduction to Topology and Modern Analysis - Tata McGraw Hill, 1963.
2. J. Dugundji : Topology - Prentice Hall of India, 1975.
3. G. J. L. Kelley, General Topology, Van Nostrand, Princeton, 1955.

## ORDINARY DIFFERENTIAL EQUATIONS

**(4 Hours/Week)**

Linear differential equations of $n^{th}$ order, fundamental sets of solutions, Wronskian–Abel's identity, theorems on linear dependence of solutions, adjoint – self - adjoint linear operator, Green's formula, Adjoint equations, the nth order non-homogeneous linear equations - Variation of parameters - zeros of solutions – comparison and separation theorems. **(15 hours)**

Fundamental existence and uniqueness theorem. Dependence of solutions on initial conditions, existence and uniqueness theorem for higher order and system of differential equations – Eigenvalue problems – Sturm-Liouville problems- Orthogonality of eigen functions - Eigen function expansion in a series of orthonormal functions- Green's functionmethod. **(15 hours)**

Power series solution of linear differential equations- ordinary and singular points ofdifferential equations, Classification into regular and irregular singular points; Series solution about an ordinary point and a regular singular point – Frobenius method- Hermite, Laguerre, Chebyshev and Gauss Hypergeometric equations and their general solutions. Generating function, Recurrence relations, Rodrigue's formula-Orthagonality properties. Behaviour of solution at irregular singular points and the point at infinity. **(15 hours)**

Linear system of homogeneous and non-homogeneous equations (matrix method) Linear and Non-linear autonomous system of equations - Phase plane - Critical points – stability - Liapunov direct method – Limit cycle and periodic solutions-Bifurcation of plane autonomous systems. **(15 hours)**

**TEXT BOOKS**

1. G.F. Simmons : Differential Equations, TMH Edition, New Delhi, 1974.
2. M.S.P. Eastham: Theory of ordinary differential equations, Van Nostrand, London, 1970.
3. S.L. Ross: Differential equations (3rd edition), John Wiley & Sons, NewYork, 1984.

**REFERENCE BOOKS**

1. E.D. Rainville and P.E. Bedient : Elementary Differential Equations, McGraw Hill,NewYork, 1969.
2. E.A. Coddington and N. Levinson : Theory of ordinary differential equations, McGrawHill, 1955.
3. A.C.King, J.Billingham and S.R.Otto: 'Differential equations', Cambridge UniversityPress, 2006.

## DISCRETE MATHEMATICS

**(4 Hours/Week)**

Introduction to logic. Methods of Proof: Rules of Inference, Valid Arguments, Rules of inference for quantified statements. Methods of proving theorems: Direct proofs, Indirect proofs, Proof by contradiction, Proof by cases. Proofs of equivalence. **(5 hours)**

Basic counting principles, the product rule and the sum rule, Examples to illustrate sum and product rule. The inclusion–exclusion principle and examples. The Pigeonhole Principle and examples.Simple arrangements and selections, Arrangement and selections with repetitions, Distributions, Binomial coefficients. **(8 hours)**

Recurrence relations, Modelling with recurrence relations with examples of Fibonacci numbers and the tower of Hanoi problem. Divide-and-Conquer relations with examples (no theorems). Generating function, definition with examples. List of generating functions. Difference equations. **(9 hours)**

Definition and types of relations. Representing relations using matrices and digraphs. Closures of relations, Paths in digraphs, Transitive closures. Warshall's Algorithm. Partial Orderings, Hasse diagrams, Maximal and Minimal elements, Lattices. **(8 hours)**

Introduction to graph theory, types of graphs, Basic terminology, Subgraphs,Representing graphs as incidence matrix and adjacency matrix. Graph isomorphism. Connectedness in simple graphs. Paths and cycles in graphs and digraphs.. Distance in graphs: Eccentricity, Radius, Diameter, Center, Periphery. Weighted graphs Dijkstra's algorithm to find the shortest distance paths in graphs and digraphs. **(9 hours)**

Euler and Hamiltonian Paths. Necessary and sufficient conditions for Euler circuits and paths in simple, undirected graphs. Hamiltonicity: noting the complexity of hamiltonicity, Traveling Salesman's Problem, Nearest neighbour method. **(7 hours)**

Distance in graphs: Eccentricity, Radius, Diameter, Center, Periphery. Planarity in graphs, Euler's Polyhedron formula. Kuratowski's theorem (statement only). Weighted graphs,Vertexconnectivity,Edgeconnectivity, covering,Independence. **(7 hours)**

Trees, Rooted trees, Binary trees, Trees as models. Properties of trees. Minimum spanning trees. Minimum spanning trees. Prims and Kruskul Algorithms. **(7 hours)**

**TEXT BOOKS**

1. C.L.Liu: Elements of Discrete Mathematics, Tata McGraw-Hill, 2000.
2. Kenneth Rosen, WCB McGraw-Hill, $6^{th}$ edition, 2004.

**REFERENCE BOOKS**

1. J.P. Tremblay and R.P. Manohar : Discrete Mathematical Structures with applications to computer science, McGraw Hill, 1975.
2. F. Harary: Graph Theory, Addition Wesley, 1969.
3. Cornelius T Leondes, Control and Dynamic systems, Academic press-2006.
4. J.H.Van Lint & RM Wilson: 'A course on combinatorics', Cambridge University Press(2006)

## PRACTICALS-I

## (Scilab and Maxima practicals and problem working)

**List of programs**

1. Introduction to Scilab and commands connected with matrices.
2. Computations with matrices.
3. Solving system of equation and explain consistence.
4. Find the values of some standard trigonometric functions in radians as well as in degree.
5. Create polynomials of different degrees and hence find its real roots.
6. Find $\sum_{n=1}^{500} n$ using looping structure.
7. Introduction to Maxima and matrices computations.
8. Commands for derivatives and $n^{th}$ derivatives.
9. Scilab and Maxima commands for plotting functions.
10. Solution of differential equation using Scilab/Maxima and plotting the solution-I.
11. Solution of differential equation using Scilab/Maxima and plotting the solution-II.

12. Solution of differential equation using Scilab/Maxima and plotting the solution-III.

**TEXT BOOKS/OPEN SOURCE MATERIALS**

1. http://maxima.sourceforge.net/docs/intromax/intromax.pdf
2. www.scilab.org.
3. wxmaxima.sourceforge.net

## PRACTICALS-II

**(Discrete mathematics practicals using Maxima or such other s\w and problem working)**

**List of programs:**

1. Logical operators: And, Or, Not, Nand, Nor, Xor, Implies, Equivalent , Unequal.
2. Finding CNF and DNF.
3. Solving recurrence/difference relations(with and without boundary conditions).
4. Finding a generating function (given a sequence).
5. Digraph representations (plotting) of relations with their properties.
6. Hasse' diagrams.
7. Lattice properties including the extremal values.
8. Graph Isomorphism (using algorithms like NAUTY)
9. Counting paths and their lengths (like Dijkstra's Algorithm)
10. Constructing Eulerian Cycles.
11. Traveling Salesman Problem.
12. Determining whether given adjacency matrix represents a tree.
13. Determining Minimum spanning trees (Prim's/ Kruskal's algorithms).

**TEXT BOOKS**

1. C. L. Liu: Elements of Discrete Mathematics, Tata McGraw-Hill, 2000.
2. Kenneth Rosen, WCB McGraw-Hill, 6$^{th}$ edition, 2004.

**REFERENCE BOOKS**

1. J.P. Tremblay and R.P. Manohar : Discrete Mathematical Structures with applications to computer science, McGraw-Hill (1975).
2. F. Harary: Graph Theory, Addition Wesley, 1969.
3. J. H. Van Lint and R. M. Wilson, "'A course on Combinatorics", Cambridge University Press (2006).
4. Allan Tucker, "Applied Combinatorics", John Wiley & Sons (1984).

## SECOND SEMESTER

## ALGEBRA-II
**(4 Hours/ Week)**

Recapitulation: Rings, Some special classes of rings (Integral domain, division ring, field, maximal and prime ideals). The prime spectrum of a ring, the nil radical and Jacobson, radical, operation on ideals, extension and contraction. **(8 hours)**

Modules - Modules and modules homomorphisms, submodules and quotient modules, Direct sums, Free modules Finitely generated modules, Nakayama Lemma, Simple modules, Exact sequences of modules. **(11 hours)**

Modules with chain conditions - Artinian and Noetherian modules, modules of finite length, Artnian rings, Noetherian rings, Hilbert basis theorem. **(11 hours)**

Extension fields**,** Finite and Algebraic extensions. Degree of extension, Algebraic elements and algebraic extensions, Adjunction of an element of a field**.** **(9 hours)**

Roots of a polynomial, Splitting fields, Construction with straight edge and compass more about roots, Simple and separable extensions, Finite fields. **(12 hours)**

Elements of Galois Theory, Fixed fields, Normal extension, Galois groups over the rationals. **(9 hours)**

**TEXT BOOKS**

1. M. F. Atiyah and I. G. Macdonald – Introduction to Commutative Algebra, Addison-Wesley. (Part A)
2. I.N. Herstein : Topics in Algebra,2nd Edition, Vikas Publishing House,1976.(Part B)

**REFERENCE BOOKS**

1. C. Musili – Introduction to Rings and Modules, Narosa Publishing House, 1997.
2. Miles Reid – Under-graduate Commutative Algebra, Cambridge University Press, 1996.

## COMPLEX ANALYSIS
**(4 Hours/ Week)**

Analytic functions, Harmonic conjugates, Elementary functions, Mobius Transformation, Conformal mappings, Cauchy's Theorem and Integral formula, Morera's Theorem, Cauchy's Theorem for triangle, rectangle, Cauchy's Theorem in a disk, Zeros of Analytic function. The index of a closed curve, counting of zeros. Principles of analytic Continuation. Liouville's Theorem, Fundaments theorem of algebra. (12 hours)

Series, Uniform convergence, Power series, Radius of convergences, Power series representation of Analytic function, Relation between Power series and Analytic function, Taylor's series, Laurent's series. (12 hours)

Rational Functions, Singularities, Poles, Classification of Singularities, Characterisation of removable Singularities, poles. Behaviour of an Analytic functions at an essential singularpoint. (8 hours)

Entire and Meromorphic functions.The Residue Theorem, Evaluation of Definite integrals, Argument principle, Rouche's Theorem, Schwartz lemma, Open mapping and Maximum modulus theorem and applications, Convex functions, Hadmard's Three circle theorem. (16 hours)

Phragmen-Lindelof theorem, The Riemann mapping theorem, Weierstrass factorization theorem. Harmonic functions, Mean Value theorem. Poisson's formula, Poisson's Integral formula, Jensen's formula, Poisson's- Jensen's formula. (12 hours)

**TEXT BOOKS**


1. J. B. Conway : Functions of one complex variable, Narosa, 1987.
2. L.V. Ahlfors : Complex Analysis, McGraw Hill, 1986.


**REFERENCE BOOKS**


1. R. Nevanlinna : Analystic functions, Springer, 1970.
2. E. Hille : Analytic Teory, Vol. I, Ginn, 1959.
3. S. Ponnaswamy : Functions of Complex variable, Narosa Publications


## FUNCTIONAL ANALYSIS
**(4 Hours/Week)**

Normed linear spaces. Banach Spaces : Definition and examples. Quotient Spaces.Convexity of the closed unit sphere of a Banach Space. Examples of normed linear spaces which are not Banach. Holder's inequality. Minkowski's inequality. Linear transformations on a normed linear space and characterisation of continuity of such transformations . (10 hours)

The set B(N,N′) of all bounded linear transformations of a normed linear space N into normed linear space N′. Linear functionals, The conjugate space N*. The natural imbedding of N into N**. Reflexive spaces. (6 hours)

Hahn -Banach theorem and its consequences, Projections on a Banach Space. The open mapping theorem and the closed graph theorem. The uniform boundedness theorem. The conjugate of an operator, properties of conjugate operator. (12 hours)

Inner product spaces, Hilbert Spaces: Definition and Examples, Schwarz's inequality.Parallelogram Law, polarization identity. Convex sets, a closed convex subset of a Hilbert Space contains a unique vector of the smallest norm. (8 hours)

Orthogonal sets in a Hilbert space. Bessel's inequality. orthogonal complements, complete orthonormal sets, Orthogonal decomposition of a Hilbert space. Characterisation of complete orthonormal set. Gram-Schmidt orthogonalization process. (8 hours)

The conjugate space H* of a Hilbert space H. Representation of a functional f as f(x)=(x, y) with y unique. The Hilbert space H*. Interpretation of T* as an operator on H. The adjoint operator T - T* on B (H). Self-adjoint operators, Positive operators. Normal operators. Unitary operators and their properties. (8 hours)

Projections on a Hilbert space. Invariant subspace. Orthogonality of projections. Eigen values and eigen space of an operator on a Hilbert Space. Spectrum of an operator on a finite dimensional Hilbert Space. Finite dimensional spectral theorem. (8 hours)

**TEXT BOOKS**

1. G. F. Simmons: Introduction to Topology and Modern Analysis(McGraw-Hill Intl. Edition).

**REFERENCE BOOKS**

1. G. Backman and L. Narici : Functional Analysis (Academic).
2. B. V. Limaye : Functional Analysis (Wiley Eastern).
3. P.R. Halmos : Finite dimensional vector paces, Van Nostrand, 1958.
4. E. Kreyszig : Introduction to Functional Analysis with Applications, John Wiley & Sons.

## TOPOLOGY-II

**(4 Hours/ Week)**

Compact spaces, Compact sets in the real line, limit point, compactness, sequential compactness and their equivalence for metric spaces. Locally Compact spaces, compactification, Alexandroff 's one point compactification. (8 hours)

The axioms of countability: First axiom space, Second countable space, Separability and the Lindelof property and their equivalence for metric spaces. (7 hours)

The product topology, the metric topology, the quotient topology, Product invariant properties for finite products, Projection maps. (7 hours)

Separation axioms: T0–space and T1 spaces –definitions and examples, the propertiesare hereditary and topological. Characterisation of T0 - and T1-spaces. (8 hours)

T2- space, unique limit for convergent sequences, Regularity and the T3–axiom. Characterisation of regularity, Metric spaces are T2 and T3. (7 hours)

Complete regularity, Normality and the T4 - axiom, Metric space is T4, compact Hausdorff space and regular lindelof spaces are normal. (8 hours)
Urysohn's Lemma, Tietze's Extension Theorem, Complete normality and the T5-axiom. (8 hours)

Local finiteness, Paracompactness, Normality of a paracompact space, Metrizability, Urysohn metrization theorem. (7 hours)

**TEXT BOOKS**

1. J.R. Munkres, Topology, 2nd Ed., Pearson Education (India), 2001.
2. W.J. Pervin : Foundations of General Topology - Academic Press, 1964.

**REFERENCE BOOKS**

1. G. F. Simmons: Introduction to Topology and Modern Analysis (McGraw-Hill International Edition).
2. G J.L. Kelley, General Topology, Van Nostrand, Princeton, 1955.
3. J. Dugundji : Topology - Prentice Hall of India, 1975

## PARTIAL DIFFERENTIAL EQUATIONS

**(4 Hours/ Week)**

First Order Partial Differential Equations:- Basic definitions, Origin of PDEs,
Classification, Geometrical interpretation. The Cauchy problem, the method of characteristics for Semi linear, quasi linear and Non-linear equations, complete integrals, Examples of equations to analytical dynamics, discontinuous solution and shockwaves. (14 hours)

Second Order Partial Differential Equations:- Definitions of Linear and Non-Linear equations, Linear Superposition principle, Classification of second-order linear partial differential equations into hyperbolic, parabolic and elliptic PDEs, Reduction to canonical forms , solution of linear Homogeneous and non-homogeneous with constant coefficients,Variable coefficients, Monge's method. (14 hours)

Wave equation: Solution by the method of separation of variables and integraltransforms The Cauchy problem, Wave equation in cylindrical and spherical polar co- ordinates. (8 hours)

Laplace equation:- Solution by the method of separation of variables and transforms. Dirchlet's, Neumann's and Churchills problems, Dirchlet's problem for a rectangle, half plane and circle, Solution of Laplace equation in cylindrical and spherical polar coordinates. (8 hours)

Diffusion equation: Fundamental solution by the method of variables and integral transforms, Duhamel's principle, Solution of the equation in cylindrical and spherical polar coordinates. (8 hours)

Solution of boundary value problems:- Green's function method for Hyperbolic, Parabolic and Elliptic equations. (8 hours)

**TEXT BOOKS**

1. N. SNEDDON, Elements of PDE's, McGraw Hill Book company Inc.
2. L. DEBNATH, Nonlinear PDE's for Scientists and Engineers, Birkhauser, Boston
3. F. John, Partial differential equations, Springer, 1971.

**REFERENCE BOOKS**


1. F. Treves: Basic linear partial differential equations, Academic Press, 1975.
2. M.G. Smith: Introduction to the theory of partial differential equations, VanNastrand, 1967
3. Shankar Rao: Partial Differential Equations, PHI


## PRACTICALS-I

**(Partial differential equations praticals and problem working)**

**List of programs:**

1. Introduction to Maxima – 2 weeks.
2. Obtaining partial derivative of some standard functions.
3. Illustrating principal of superposition for linear partial differential equation.
4. Classification of $2^{nd}$ order PDE' s into parabola, elliptic and hyperbola.
5. Obtaining the solution of wave equation by Fourier decomposition method (Separation of variables).
6. Plotting of the double Fourier series solutions for wave equation and discussing about convergence.
7. Obtaining the solution of wave equation by Fourier transforms.
8. Obtaining the solution of Laplace equation by Fourier decomposition method (Separation of variables).
9. Obtaining the solution of Laplace equation by Fourier transforms.
10. Obtaining the solution of Heat equation by Fourier decomposition method (Separation of variables).
11. Obtaining the solution of Heat equation by Fourier transforms.
12. Implementing the green's function method for hyperbolic PDE.

**TEXT BOOKS / OPEN SOURCE MATERIALS**


1. I. N. SNEDDON, Elements of PDE's , McGraw Hill Book company Inc. 2009
2. L DEBNATH , Nonlinear PDE's for Scientists and Engineers, Birkhauser , Boston, 2008.
3. F. John, Partial differential equations, Springer, 1971.
4. wxmaxima.sourceforge.net


**REFERENCE BOOKS**


1. F. Treves: Basic linear partial differential equations, Academic Press, 1975.
2. M.G. Smith: Introduction to the theory of partial differential equations, Van Nostrand, 1967
3. Shankar Rao: Partial Differential Equations, PHI, 2009.

## PRACTICALS-II
### (Numerical Analysis - I Practicals and problem working)

**List of programs:**

1. Introduction to Scilab – 2 weeks
2. Fixed Point iterative method
3. Newton-Raphson’s method
4. Ramanujan’s method
5. Gauss Elimination method
6. Gauss-Seidel iterative method
7. Thomas Algorithm
8. Lagrange Interpolation method
9. Cubic Spline Interpolation method
10. Rational function approximation of Pade Numerical integration over rectangular region
11. Gaussian Quadrature method
12. Gauss-Chebyshev method

**TEXT BOOKS/ OPEN SOURCE MATERIALS**

1. M.K. Jain: Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.
2. C.F. Gerald and P.O. Wheatley : Applied Numerical Methods, Low- priced edition, Pearson Education Asia (2002), Sixth Edition.
3. D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Blackwell Scientific Publications (1991).
4. www.scilab.org

**REFERENCE BOOKS**

1. S.C. Chapra, and P.C. Raymond : Numerical Methods for Engineers, Tata Mc Graw Hill, New Delhi (2000)
2. R.L. Burden, and J. Douglas Faires : Numerical Analysis, P.W.S. Kent Publishing Company, Boston (1989), Fourth edition.
3. S.S. Sastry : Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi (1998).
4. M.K. Jain, S.R.K. Iyengar and R.K. Jain : Numerical methods for scientific and Engineering computation, Wiley Eastern (1993)
5. G.D.Smith: Numerical Solutions of partial differential equations $2^{nd}$ edition London, Oxford University Press (1978)
6. Paruiz Moin: Fundamentals of Engineering Numerical analysis, Cambridge University Press (2006)
7. SCILAB- A Free software to MATLAB by Er. Hema Ramachandran and Dr. Achuthsankar S. Nair., S. Chand and Company Ltd. (2008)

# THIRD SEMESTER

## DIFFERENTIAL GEOMETRY
**(4 Hours/week)**

Calculus on Euclidean Space: Euclidean space. Natural coordinate functions. Differentiable functions. Tangent vectors and tangent spaces. Vector fields. Directional derivatives and their properties. Curves in $E^3$. Velocity and speed of a curve. Reparametrization of a curve. 1-forms and Differential forms. Wedge product of forms. Mappings of Euclidean spaces. Derivative map. (15 hours)

Frame Fields:Arc length parametrization of curves. Vector field along a curve. Tangent vector field, Normal vector field and Binormal vector field. Curvature and torsion of a curve. The Frenet formulas Frenet approximation of unit speed curve and Geometrical interpretation. Properties of plane curves and spherical curves. Arbitrary speed curves. Cylindrical helix Covariant derivatives and covariant differentials. Cylindrical and spherical frame fields. Connection forms. Attitude matrix. Structural equations. Isometries of $E^3$- Translation, Rotation and Orthogonaltransformation.The derivative map of an isometry. (15 hours)

Calculus on a Surface: Coordinate patch. Monge patch. Surface in $E^3$ . Special surfaces- sphere, cylinder and surface of revolution. Parameter curves, velocity vectors of parameter curves, Patch computation. Parametrization of surfaces- cylinder, surface of revolution and torus. Tangent vectors, vector fields and curves on a surface in $E^3$. Directional derivative of a function on a surface of $E^3$. Differential forms and exterior derivative of forms on surface of $E^3$. Pull back functions on surfaces of $E^3$ . (15 hours)

Shape Operators: Definition of shape operator. Shape operators of sphere, plane, cylinder and saddle surface. Normal curvature, Normal section. Principal curvature and principal direction. Umbilic points of a surface in $E^3$. Euler's formula for normal curvature of a surface in $E^3$. Gaussian curvature, Mean curvature and Computational techniques for these curvatures. Minimal surfaces. Special curves in a surface of $E^3$ -Principal curve, geodesic curve and asymptotic curves. Special surface - Surface of revolution. (15 hours)

**TEXT BOOKS**

1. Barrett O' Neil : Elementary Differential Geometry. Academic Press, New York and London, 1966
2. T.J.Willmore : An introduction to Differential Geometry. Clarendon Press, Oxford 1959.

**REFERENCE BOOKS**

1. D.J.Struik : Lectures on Classical Differential Geometry, Addison Wesley, Reading, Massachusetts, 1961.
2. Nirmala Prakassh: Differential Geometry- an integrated approach. Tata McGraw-Hill, New Delhi, 1981.

## FLUID MECHANICS
**(4 Hours/week)**

Coordinate transformations - Cartesian tensors - Basic Properties - Transpose - Symmetric and Skew tensors - Isotropic tensors- Deviatoric Tensors - Gradient, Divergence and Curl in Tensor Calculus - Integral Theorems.
Continuum Hypothesis- Configuration of a continuum – Mass and density – Description of motion – Material and spatial coordinates - Translation – Rotation - Deformation of a surface element- Deformation of a volume element - Isochoric deformation – Examples - Stretch and Rotation- Decomposition of a deformation- Deformation gradient - Strain tensors - Infinitesimal strain - Compatibility relations - Principal strains.
Material and Local time derivatives.- Strain-rate tensor- Transport formulas – Stream lines - Path lines - Vorticity and Circulation - Examples.
Stress components and Stress tensor - Normal and shear stresses - Principal stresses. Fundamental basic physical laws- Law of conservation of mass - Principle of linear and momentum - Balance of energy - Examples.
Equations of fluid mechanics – Viscous and non-viscous fluids –Stress tensor for a viscous fluid – Navier-Stokes equation - simple consequences and simple applications.

(**28 hours )**

Motion of inviscid fluids:- Recapitulation of equation of motion and standard results - Vortex motion- Helmholtz vorticity equation - Permanence of vorticity and circulation - Kelvin's minimum energy theorem – Impulsive motion - Dimensional analysis - Nondimensional numbers.
Two dimensional flows of inviscid fluids:- Meaning of two-dimensional flow - Stream function – Complex potential - Line sources and sinks - Line doublets and vortices - Images - Milne-Thomson circle theorem and applications - Blasius theorem and applications.

**(16 hours )**

Motion of Viscous fluids:- Stress tensor – Navier-Stokes equation - Energy equation - Simple exact solutions of Navier-Stokes equation: (i) Plane Poiseuille and Hagen- Poiseuille flows (ii) Generalized plane Couette flow (iii) Steady flow between tworotating concentric circular cylinders (iv) Stokes's first and second problems. Diffusion of vorticity - Energy dissipation due to viscosity.

**(16 hours )**

### TEXT BOOKS
1. D.S. Chandrasekharaiah and L. Debnath: Continuum Mechanics, Academic Press, 1994.
2. A.J.M. Spencer: Continuum Mechanics, Longman, 1980.
3. S. W. Yuan : Foundations of Fluid Mechanics, Prentice Hall, 1976.

### REFERENCE BOOKS
1. P. Chadwick : Continuum Mechanics, Allen and Unwin, 1976.
2. L.E. Malvern : Introduction to the Mechanics of a Continuous Media,
Prentice Hall,1969.
3. Y.C. Fung, A First course in Continuum Mechanics, Prentice Hall
(2nd edition), 1977.
4. Pijush K. Kundu, Ira M. Cohen and David R. Dowling, Fluid Mechanics, Fifth Edition , 2010.
5. C.S.Yih : Fluid Mechanics, McGraw-Hill, 1969.

## COMPUTATIONAL LINEAR ALGEBRA
**(4 Hours/week)**

Recapitulation: Vector Spaces, Subspaces, Linear Combinations and Systems of Linear Equations, Linear dependence and independence, Basis and dimension, Maximal Linearly independence subsets, Direct sums, Linear Transformation and Linear Operators. (4 Hours)

Algebra of Linear Transformation, Minimal Polynomial, Regular and Singular Transformation, Range and rank of a transformation and its properties, characteristics roots and characteristics vectors. (10 Hours)

The matrix representation of a linear transformation, Composition of a linear transformation and matrix multiplication, The change of coordinate matrix, transition matrix, The dual space. (8 Hours)

Characteristic polynomials, Diagonalisability, Invariant Subspace, Cayley-Hamilton Theorem. (8 Hours)

Canonical forms, Triangular canonical forms, Nilpotent transformation, Jordan Canonical form, The rational canonical form. (8 Hours)

Positive Definite Matrices, Maxima, minima and saddle points, Test for positive definiteness, Singular value decomposition and its application. (8 Hours)

Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, rank and signature, Sylvester's law of inertia. (8 Hours)

**TEXT BOOKS**


1. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education, 2003, Prentice-Hall of India, 1991.
2. N. Herstein, Topics, in Algebra, 2nd Ed., John Wiley & Sons, 2006.
3. S. Freidberg, A. Insel and L. Spence: Linear Algebra, 4th Ed., PHI.
4. J. Gilbert and L. Gilbert, Linear Algebra and Matrix Theory, 1995.


**REFERENCE BOOKS**


1. S. Lang, Linear Algebra, Springe-Verlag, New York, 1989.
2. M. Artin, Prentice-Hall of India, 1994.
3. G. Strang, Linear Algebra and its Application, Brooks/Cole Ltd., New Delhi, 3rd Ed., 2003.
4. L. Hogben, Handbook of Linear Algebra, Chapman and Hall, CRC, 2006.

## DEPARTMENTAL ELECTIVE

**Choose any one of the following papers**
**A. MATHEMATICAL METHODS**
**B. NUMERICAL ANALYSIS**

### MTDE 9415: MATHEMATICAL METHODS 30 Hours

Integral Equations: Definition, Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series resolvent kernel and transform methods, Convergence for Fredholm and Volterra types. Reduction of IVPs BVPs and eigenvalue problems to integral equations. Hilbert Schmidt theorem, Raleigh Ritz and Galerkin methods. (16 hours)

Asymptotic expansions: Asymptotic expansion of functions, power series as asymptotic series, Asymptotic forms for large and small variables. Uniqueness properties and Operations.
Asymptotic expansions of integrals; Method of integration by parts (include examples where the method fails), Laplace's method and Watson's lemma, method of stationary phase and steepest descent. (14 hours)

### MTDE 9515: NUMERICAL ANALYSIS 30 Hours

Runge Kutta methods of first, second and fourth order- stability-(Convergence and Truncation error for the above methods). Multistep method- The Adam-Bashfort-Moulton method of second and fourth orders. Boundary- Value problems-Second order finite difference methods. (12 hours)

Numerical solution of Partial differential equations: Difference methods for Elliptic partial differential equations – Difference schemes for Laplace and Poisson's equations. Iterative methods of solution by Jacobi and Gauss Siedel methods – solution techniques for rectangular and quadrilateral regions. Difference methods for parabolic equations in one-dimension – methods of Schmidt and Crank-Nicolson. Stability and convergence analysis for Schmidt and Crank-Nicolson methods-A.D.I. method for two-dimensional parabolic equation. Explicit finite difference schemes for hyperbolic equations- wave equation in one-dimension. (18 hours)

### TEXT BOOKS


1. I.N.Sneddon- The use of Integral Transforms, Tata Mc Graw Hill, Publishing Company Ltd, New Delhi, 1974.
2. R.P.Kanwal: Linear integral equations theory and techniques, Academic Press, New York, 1971.
3. C.M.Bender and S.A. Orszag-Advanced mathematical methods for scientists and engineers, Mc Graw Hill, New York, 1978.

## PRACTICALS-I
## Computational Linear Algebra Practicals and problem working

(2+2 Hrs/week) Softwares such as GeoGebra, Maxima, Scilab or any other FOSS will be used)

**List of Programs**

1. Linear independence, Linear combinations, Change of basis
2. Linear transformation to matrices conversion and vice versa
3. Matrix with respect to change of basis
4. Finding eigen values and their multiplicity as roots of $\det(A-\lambda I)=0$.
5. Calculation of as many as linear independent eigen vectors(eigen spaces)
6. Calculation of eigen values and eigen vectors for a symmetric matrix
7. Orthogonal and orthonormal sets
8. Eigen values and orthonormal eigen vectors
9. Gram-Schmidt orthogonalization of the columns
10. Triangularisation
11. Diagonalisation
12. Singular value decomposition

**TEXT BOOKS**


1. K. Hoffman and R. Kunze, Linear Algebra, Pearson Education (India) 2003. Prentice –Hall of India, 1991.
2. I.N. Herstein: Topics in Algebra, 2nd Edition, John Wiley &Sons, 2006.


## PRACTICALS-II
## Mathematical Methods

**List of Programs:**

1. To find the solution of given IBVP using Schmidt method for one-dimensional heat equation.
2. To find the solution of IBVP using Crank Nicholson method for one-dimensional heat equation.
3. To find solution of one-dimensional wave equation by using explicit finite difference method.
4. To find solution of given ODE by using Runge-Kutta method of 2$^{nd}$ order.
5. To find the solution of given ODE by using Runge-Kutta method of 4$^{th}$ order.
6. To find solution of two-dimensional Laplace equations by using finite difference method.
7. To find the solution of two-dimensional Poisson equations by using finite difference method.
8. To find the value of integral equations as $x \to \infty$ and compare leading order solution obtained by Laplace method.
9. To find the value of integral equations as $x \to 0$.
10. To verify Watson Lemma with numerical integration

# FOURTH SEMESTER

## MEASURE AND INTEGRATION

**(4 Hours/week)**

Algebra of sets, sigma algebras, open subsets of the real line. $F_\sigma$ and $G_\delta$ sets, Borel sets, Outer measure of a subset of R 'Lebesgue outer measure of a subset of R Existence, non-negativity and monotonicity of Lebesgue outer measure; Relation between Lebesgue outer measure and length of an interval; Countable subadditivity of Lebesgue outer measure; translation invariance**.** (10 hours)

(Lebesgue) measurable sets, (Lebesgue) measure; Complement, union, intersection and difference of measurable sets; denumerable union and intersection of measurable sets; countable additivity of measure; The class of measurable sets as a algebra, the measure of the intersection of a decreasing sequence of measurable sets. (9 hours)

Measurable functions; Scalar multiple, sum, difference and product of measurable functions.Measurability of a continuous function and measurability of a continuous image of measurable function.Convergence pointwise and convergence in measures of a sequence of measurable functions. (8 hours)

Lebesgue Integral; Characteristic function of a set; simple function; Lebesgue integral of a simple function; Lebesgue integral of a bounded measurable function; Lebesgue integral and Riemann integral of a bounded function defined on a closed interval; Lebesgue integral of a non-negative function; Lebesgue integral of a measurable function; Properties of Lebesgue integral. (7 hours)

Convergence Theorems and Lebesgue integral; The bounded convergence theorem; Fatou's Lemma: Monotone convergence theorem; Lebesgue convergence theorem. (7 hours)

Differentiation of Monotone functions. Vitali covering lemma.Functions of Bounded variation.Differentiability of an integral.Absolute continuity and indefinite integrals. (9 hours)

Lp spaces. Holder and Minkowski inequalities.Convergence and completeness, Riesz – Fischer Theorem.Bounded linear functionalsRiesz representation theorem and illustrative examples. Measure spaces, Signed measures, the Radon Nikodyn theorem. (9 hours)

**TEXT BOOKS**

1. H.L. Royden : Real Analysis, Macmillan, 1963

**REFERENCE BOOKS**

1. P.R. Halmos : Measure Theory, East West Press, 1962
2. W. Rudin : Real & Complex Analysis, McGraw Hill , 1966

# OPTIONAL PAPER-I

**Choose any one of the following papers**
**A. RIEMANNIAN GEOMENTRY**
**B. SPECIAL FUNCTIONS**
**C. THEORY OF NUMBERS**
**D. ENTIRE AND MEROMORPHIC FUNCTIONS**

## RIEMANNIAN GEOMENTRY
**(4 Hours/week)**

Differentiable manifolds:- Charts, Atlases, Differentiable structures, Topology induced by differentiable structures, equivalent atlases, complete atlases. Manifolds. Examples of manifolds. Properties of induced topology on manifolds. (10 hours)

Tangent and cotangent spaces to a manifold. Vector fields. Lie bracket of vector fields. Smooth maps and diffeomorphism. Derivative(Jacobi) of smooth maps and their matrix representation. Pull back functions (10 hours)

Tensor fields and their components. Transformation formula for components of tensors. Operations on tensors. Contraction, Covariant derivatives of tensor fields. (10 hours)

Riemannian Metric. Connections. Riemannian connections and their components, Parallel translation, Fundamental theorem of Riemannian Geometry. Curvature and torsion tensors. Bianchi identities, Curvature tensor of second kind. Sectional curvature. Space of constant curvature. Schur's theorem. (10 hours)

Curves and geodesics in Riemannian manifold. Geodesic curvature, Frenet formula. (10 hours)

Hypersurfaces of Riemannian manifolds Gauss formula, Gauss equation, Codazzi equation, Sectional curvature for a hyper surface of a Riemannian manifold, Gauss map, Weingartan map and Fundamental forms on hypersurface. Equations of Gauss and Codazzi. Gauss theorem egregium. (10 hours)

**TEXT BOOKS**


1. Y. Matsushima : Differentiable manifolds. Marcel Dekker Inc. New, York,1972.
2. W.M .Boothby : An introduction to differentiable manifolds and Riemannian Geometry.Academic Press Inc. New York, 1975.
3. N.J. Hicks : Notes on differential Geometry D.Van Nostrand company Inc. Princeton, New Jersey, New York, London (Affiliated East-West Press Pvt. Ltd. New Delhi), 1998.


**REFERENCE BOOKS**


1. R.L. Bishop and Grittendo : Geometry of manifolds. Acamedic Press, New York, 1964.
2. L.P. Eisenhart : Riemannian Geometry. Princeton University Press, Princetion, New Jersey, 1949.
3. H. Flanders : Differential forms with applications to the physical science,

Academic Press, New York, 1963.

4. R.L. Bishop and S.J. Goldberg : Tensor analysis on manifolds, Macmillan Co., 1968.
5. K. S. Amur, D.J. Shetty and C. S. Bagewadi, An introduction to differential Geometry, Narosa Pub. New Dehli, 2010.

## SPECIAL FUNCTIONS
### (4 Hours/week)

Hypergeometric series: Definition- convergence- Solution of second order ordinary differential equation or Gauss equation- Confluent hypergeometric series- Binomial theorem, Integral Representation- Gauss's Summation formula- Chu-Vandermonde Summation formula-Pfaff-Kummer Transformation Formula-Euler's transformation formula. (12 hours)

Basic-hypergeometric series: Definition- Convergence- q− binomial theorem- Heines transformation formula and its q-analogue- Jackson transformation formula- Jacobi's triple product identity and its applications (proof as in ref. 9)- Quintuple product identity (proof as in reference 10)- Ramanujan's 1 ψ1 summation formula and its applications- A new identity for with an application to Ramanujan partition congruence modulo 11- Ramanujan theta-function identities involving Lambert series. 10(;)*qq*∞ (16 hours)

q−series and Theta-functions: Ramanujan's general theta-function and special cases- Entries 18, 21, 23, 24, 25, 27, 29, 30 and 31 of Ramanujan's Second note book (as in text book reference 4). (10 hours)

Partitions: Definition of partition of a +ve integer- Graphical representation- Conjugate- Self-conjugate- Generating function of p (n)- other generating functions- A theorem of Jacobi- Theorems 353 and 354- applications of theorem 353- Congruence properties of p (n)- $p(5n + 4) \equiv 0 \pmod 5$ and $p(7n + 4) \equiv 0 \pmod 7$- Two theorems of Euler- Rogers-Ramanujan Identities- combinatorial proofs of Euler's identity, Euler's pentagonal number theorem. Franklin combinatorial proof. Restricted partitions- Gaussian. (portion to be covered as per Chapter-XIX of An Introduction to the Theory of Numbers written by G. H. Hardy and E. M. Wright). (22 hours)

**TEXT BOOKS**

1. C. Adiga, B. C. Berndt, S. Bhargava and G. N. Watson, Chapter 16 of Ramanujan's second notebook: Theta-function and q-series, Mem. Amer. Math. Soc., 53, No.315 ,Amer. Math. Soc., Providence, 1985.
2. T. M. Apostol: Introduction to Analytical number theory, Oxford University Press, 2000.
3. G. E. Andrews, The theory of Partition, Cambridge University Press, 1984
4. B. C. Berndt, Ramanujans notebooks, Part-III, Springer-Verlag, New York, 1991.
5. B. C. Berndt, Ramanujan's notebooks, Part-IV, Springer-Verlag, New York,1994
6. B. C. Berndt, Ramanujans notebooks, Part-V, Springer-Verlag, New York, 1998
7. George Gasper and Mizan Rahman, Basic hyper-geometric series, Cambridge University Press, 1990.
8. G. H. Hardy and E. M. Wright, An Introduction of the Theory of Numbers, Oxford University Press, 1996.

## REFERENCE BOOKS


1. B. C. Berndt, S. H. Chan, Zhi-Guo Liu, and Hamza Yesilyurt, A new identities for with an application to Ramanujan partition congruence modulo 11, Quart. J. Math. 55,13-30, 2004,. 10(;)*qq*∞
2. M. S. Mahadeva Naika and H. S. Madhusudhan, Ramanujan's Theta-function identities involving Lambert Series, Adv. Stud. Contemp. Math., 8, No.1, 3-12, MR 2022031 (2004j: 33021), 2004.
3. M. S. Mahadeva Naika and K. Shivashankara, Ramanujan's summation formula and related identities, Leonhard Paul Euler Tricentennial Birthday Anniversary Collection, J. App. Math. Stat., 11(7), pp. 130-137, 2007. 11Ψ
4. Sarachai Kongsiriwong and Zhi-Guo Liu, Uniform proofs of q-series-product identity, Result. Math., 44(4), pp. 312-339, 2003.
5. Shaun Cooper, The Quintuple product identity, International Journal of Number Theory, Vol. 2(1), 115-161, 2006.


## THEORY OF NUMBERS
**(4 Hours/week)**

Multiplicative and completely multiplicative functions. Euler Toteint function. Möbius and Mangoldt function. Dirichlet product and the group of arithmetical function. Generalised convolution. Formal power series. Bell series. (18 hours)

Residue Classes and complete Residue Classes, Linear Congruences an Euler-Fermat Theorem, General Polynomial congruences and Lagrange Theorem, Wilson's Theorem, Chinese Reminder Theorem. Fundamental Theorem on Polynomial Congruences with prime power moduli. Quadratic Residue and Gauss's Law of Quadratic Reciprocity. (both for Legendre and Jacobi symbols) Primitive roots and their existence for moduli m=1, 2, 4, pα, 2pα. (21 hours)

Partition: partition of a +ve integer, Graphical representation, Conjugate, Generating functions, A theorem of Jacobi, Theorem 353 and 354, Applications of theorem 353. Congruence properties of P(n), Two theorems of Euler, Rogers – Ramanujan Identities (portion to be covered as per Chapter-XIX of "An Introduction to the Theory of Numbers" written by G. H. Hardy and E. M. Wright.). (21 hours)

## TEXT BOOKS


1. T. M. Apostol: Introduction to Analytical number theory, Oxford University Press, 2000.
2. G. H. Hardy and E. M. Wright: An introduction to the Theory of Numbers, Oxford University Press, 1996.
3. Thomas Keshy: Elementary Number Theory with Applications Acad Press, 2005.


## REFERENCE BOOKS


1. I. Niven and H. S. Zuckerman: An introduction to the Theory of Numbers, John Wiley, 2002.
2. J. V. Uspensky and M. A. Heaslott: Elementary Number Theory, Mc Graw-Hill 1996.

## ENTIRE AND MEROMORPHIC FUNCTIONS
**(4 Hours/week)**

Basic properties of Entire Functions. Order and Type of an Entire Functions . Relationship between the Order of an Entire Function and its Derivative . Exponent of Convergence of Zeros of an Entire Function. Picard and Borel's Theorems for Entire Functions. (16 hours)

Asymptotic Values and Asymptotic Curves. Connection between Asymptotic and various Exceptional Values. (8 hours)

Meromorphic Functions. Nevanlina's Characteristic Function. Cartan's Identity and Convexity Theorems. Nevanlinna's First and Second Fundamental Theorems .Order and Type of a Meromorphic Function. Order of a Meromorphic Function and its Derivative. Relationship between T(r, f) and log M(r, f) for an Entire Function. Basic properties of T(r, f). (18 hours)

Deficient Values and Relation between various Exceptional Values. Fundamental Inequality of Deficient Values. Some Applications of Nevanlinna's Second Fundamental theorem. Functions taking the same values at the same points. Fix-points of Integral Functions. (18 hours)

**TEXT BOOKS**

1. A. I. Markushevich: Theory of Functions of a complex Variables, Vol.-II, Prentice-Hall, (1965).
2. A. S. B. Holland : Introduction to the theory of Entire Functions, Academic Press,New York, (1973).

**REFERENCE BOOKS**

1. C. L. Siegel: Nine Introductions in Complex Analysis, North Holland, (1981)
2. W. K. Hayman : Meromorphic Functions ,Oxford University, Press, (1964).
3. Yang La : Value Distribution Theory, Springer Verlag, Scientific Press, (1964).
4. Laine : Nevanlinxa theory and Complex Differerntial Equations, Walter de Gruyter, Berlin (1993).

## OPTIONAL PAPER-II

**Choose any one of the following papers**

A. **MAGNETOHYDRODYNAMICS**
B. **FLUID DYNAMICS OF OCEAN AND ATMOSPHERE**
C. **COMPUTATIONAL FLUID DYNAMICS**

## MAGNETOHYDRODYNAMICS
**(4 Hours/week)**

Electrodynamics: Electrostatics and electromagnetic units –derivation of Gauss law- Faraday’s law- Ampere’s law and solenoidal property-–conservation of charges-electromagnetic boundary conditions. Dielectric materials. (14 hours)

Basic Equations: Derivation of basic equations of MHD - MHD approximations - Non-dimensional numbers – Boundary conditions on velocity, temperature and magnetic. (8 hours)

Classical MHD: Alfven’s theorem- Frozen-in-phenomenon-illustrative examples-Kelvin’s circulation theorem-Bernoulli’s equations-Analogue of Helmholtz vorticity equation-Ferraro’s law of isorotation. (8 hours)

Magnetostatics: Force free magnetic field and important results thereon-illustrative examples on abnormality parameter-Chandrasekhar’s theorem-Bennett pinch and instabilities associated with it. (8 hours)

Alfven waves: Lorentz force as a sum of two surface forces- cause for Alfven waves- applications- Alfven wave equations in incompressible fluids- equipartition of energy – experiments on Alfven waves- dispersion relations- Alfven waves in compressible fluids- slow and fast waves-Hodographs. (14 hours)

Flow Problems: Hartmann flow- Hartmann –Couette flow- Temperature distribution for these flows. (8 hours)

### TEXT BOOKS

1. T.G.Cowling : Magnetohydrodynamics, Interscience, 1957.
2. V.C.A.Ferraro and C.Plumpton : An Introduction to Magneto-Fluid Mechanics,Oxford University Press, 1961.
3. G.W.Sutton and A.Sherman : Engineering Magnetohydrodynamics, McGraw Hill, 1965.
4. Alan Jeffrey : Magnetohydrodynamics, Oliver & Boyd, 1966.
5. K.R.Cramer and S.I.Pai : Magnetofluid Dynamics for Engineers and Applied Physicists, Scripta Publishing Company, 1973.

### REFERENCE BOOKS

1. D.J.Griffiths : Introduction to Electrohydrodynamics, Prentice Hall, 1997.
2. P.H.Roberts : An Introduction to Magnetohydrodynamics, Longman, 1967.
3. H.K.Moffat : Magnetic field generation in electrically conducting fluids,Cambridge University Press, 1978.

## FLUID DYNAMICS OF OCEAN AND ATMOSPHERE
**(4 Hours/week)**

Introduction:- Fundamental concepts – Density stratification – Equations of Motion in a rotating Coordinate frame – Coriolis acceleration, Circulation – Vorticity equation – Kelvin’s theory – potential vorticity (standard results) – Thermal wind – Geostrophic motion. Hydrostatic approximation, Consequences. Taylor - Proudman theorem Geostrophic Degeneracy. Dimensional analysis and nondimensional numbers.

(12 hours)

Physical Meteorology:- Atmospheric composition, laws of thermodynamics of the atmosphere, adiabatic process, potential temperature. The Clausius Clapyeron equation. Laws of black body radiation, solar and terrestrial radiation, solar constant, Albedo, greenhouse effect, heat balance of earth- atmosphere system. (12 hours)

Atmospheric Dynamics :- Geostrophic approximation. Pressure as a vertical oordinate. Modified continuity equation. Balance of forces. Non-dimensional numbers (Rossby, Richardson, Froude, Ekman etc). Scale analysis for tropics and extra- tropics, vorticity and divergence equations, conservation of potential vorticity. Atmospheric turbulence and equations for planetary boundary layer. (12 hours)

Homogeneous Models of the wind-driven Oceanic circulation:- The Homogeneous model – The Sverdrup relation. General Circulation of the Atmosphere :- Definition of the general circulation, various components of the general circulation – zonal and eddy angular momentum balance of the atmosphere, meridional circulation, Hadley Ferrel and polar cells in summer and winter, North-South and East- West (Walker) monsoon circulation. Forces meridional circulation due to heating and momentum transport. Available potential energy, zonal and eddy energy equations. (12 hours)

Atmospheric Waves and Instability :- Wave motion in general. Concept of wave packet, phase velocity and group velocity. Momentum and energy transports by waves in the horizontal and vertical directions. Equatorial, Kelvin and mixed Rossby gravity waves. Stationary planetary waves. Filtering of sound and gravity waves. Linear barotropic and baroclinic instability. (12 hours)

**TEXT BOOKS**


1. Joseph Pedlosky : Geophysical fluid Dynamics, Springer, Second Edition, 1987
2. G.K. Batchelor : An introduction to fluid Dynamics, Cambridge University Press,1967
3. H. Schlichting : Boundary layer theory, Mc Graw Hill, 1968
4. A. Defant : Physical Occanognaphy , Vol.1 Pergamon Press, 1961
5. J.D. Cole : Perturbation methods in applied mathematics, Blaisedell, 1968


**REFERENCE BOOKS**


1. M. Van Dyke: Perturbation methods in fluid mechanics, Acad, Press, 1964
2. J.R. Holton : An introduction to Dynamic Meteorology, Acad. Press, 1991.
3. Ghill and Childress: Topics in Geophysical Fluid Dynamics, Applied Mathematical Science, Springer Verlag, 1987
4. E. E. Gossard and W.H. Hooke : Waves in the Atmosphere, Elsevier, 197538
5. John Houghton: The Physics of Atmospheres, Cambridge University Press (3$^{rd}$ edition), 2002


## COMPUTATIONAL FLUID DYNAMICS (CFD)
**(4 Hours/week)**

Review of partial differential equations, numerical analysis, fluid mechanics. (6 hours)

Finite Difference Methods: Derivation of finite difference methods, finite difference method to parabolic, hyperbolic and elliptic equations, finite difference method to nonlinear equations, coordinate transformation for arbitrary geometry, Central schemes with combined space-time discretization-Lax-Friedrichs, Lax-Wendroff, MacCormack methods, Artificial compressibility method, pressure correction method – Lubrication model, Convection dominated flows – Euler equation – Quasilinearization of Euler equation, Compatibility relations, nonlinear Burger equation. (20 hours)

Finite Volume Methods: General introduction, Node-centered-control volume, Cell-centered-control volume and average volume, Cell-Centred scheme, Cell-Vertex scheme, Structured and Unstructured FVMs, Second and Fourth order approximations to the convection and diffusion equations (One and Two-dimensional examples). (14 hours)

Finite Element Methods: Introduction to finite element methods, one-and two-dimensional bases functions – Lagrange and Hermite polynomials elements, triangular and rectangular elements, Finite element method for one-dimensional problem: model boundary value problems, discretization of the domain, derivation of elemental equations and their connectivity, composition of boundary conditions and solutions of the algebraic equations. Finite element method for two-dimensional problems: model equations, discretization, interpolation functions, evaluation of element matrices and vectors and their assemblage. (20 hours)

**TEXT BOOKS**

1. T. J. Chung: 'Computational Fluid Dynamics', Cambridge Univ. Press, 2003.
2. J Blazek, 'Computational Fluid Dynamics', Elsevier, 2001.
3. Harvard Lomax, Thomas H. Pulliam, David W Zingg, 'Fundamentals of Computational Fluid Dynamics', NASA Report, 2006.

**REFERENCE BOOKS**

1. C.A J. Fletcher: 'Computational techniques for Fluid Dynamics', Vol. I & II, Springer Verlag 1991.

## OPTIONAL PAPER-III

**Choose any one of the following papers**
**A. FINITE ELEMENT METHOD WITH APPLICATION**
**B. GRAPH THEORY**
**C. DESIGN AND ANALYSIS OF ALGORITHMS**

## FINITE ELEMENT METHOD WITH APPLICATIONS
**(4 Hours/week)**

Weighted Residual Approximations:- Point collocation, Galerkin and Least Squares method. Use of trial functions to the solution of differential equations. (12 hours)

Finite Elements:- One dimensional and two dimensional basis functions,Lagrange and serendipity family elements for quadrilaterals and triangular shapes. Isoparametric coordinate transformation. Area coordinates standard 2- squares and unit triangles in natural coordinates. (16 hours)

Finite Element Procedures:- Finite Element Formulations for the solutions of ordinary and partial differential equations: Calculation of element matrices, assembly and solution of linear equations. (16 hours)

Finite Element solution of one dimensional ordinary differential equations, Laplace and Poisson equations over rectangular and nonrectangular and curved domains. Applications to some problems in linear elasticity: Torsion of shafts of a square, elliptic and triangular cross sections. (16 hours)

**TEXT BOOKS**


1. O.C. Zienkiewiez and K. Morgan : Finite Elements and approximation, John Wieley, 1983
2. P.E. Lewis and J.P. Ward : The Finite element method- Principles and applications, Addison Weley, 1991
3. L.J. Segerlind : Applied finite element analysis (2nd Edition), John Wiley, 1984


**REFERENCE BOOKS**


1. O.C. Zienkiewicz and R.L.Taylor : The finite element method. Vol.1 Basic formulation and Linear problems, 4th Edition, New York, Mc. Graw Hill, 1989.
2. J.N. Reddy: An introduction to finite element method, New York, Mc.Graw Hill, 1984.
3. D.W. Pepper and J.C. Heinrich : The finite element method, Basic concepts and applications, Hemisphere, Publishing Corporation, Washington, 1992.
4. S.S. Rao : The finite element method in Engineering, 2nd Edition, Oxford, Pergamon Press, 1989.
5. D. V. Hutton, fundamental of Finite Element Analysis, (2004).
6. E. G. Thomson, Introduction to Finite Elements Method, Theory Programming and applications, Wiley Student Edition, (2005).

# GRAPH THEORY
**(4 Hours/week)**

Connectivity :- Cut- vertex, Bridge, Blocks, Vertex-connectivity, Edge-connectivity and some external problems, Mengers Theorems, Properties of n-connected graphs with respect to vertices and edges. (9 hours)

Planarity:- Plane and Planar graphs, Euler Identity, Non planar graphs, Maximal planar graph Outer planar graphs, Maximal outer planar graphs, Characterization of planar graphs , Geometric dual, Crossing number. (9 hours)

Colorability :- Vertex Coloring, Color class, n-coloring, Chromatic index of a graph, Chromatic number of standard graphs, Bichromatic graphs, Colorings in critical graphs, Relation between chromatic number and clique number/independence number/maximum degree, Edge coloring, Edge chromatic number of standard graphs Coloring of a plane map, Four color problem, Five color theorem, Uniquely colorable graph. Chromatic polynomial. (12 hours)

Matchings and factorization:-Matching- perfect matching, augmenting paths, maximum matching, Hall's theorem for bipartite graphs, the personnel assignment problem, a matching algorithm for bipartite graphs, Factorizations, 1-factorization, 2-factorization. Partitions-degree sequence, Havel's and Hakimi algorithms and graphical related problems. (12 hours)

Directed Graphs:- Preliminaries of digraph, Oriented graph, indegree and outdegree, Elementary theorems in digraph, Types of digraph, Tournament, Cyclic and transitive tournament, Spanning path in a tournament, Tournament with a hamiltonian path, strongly connected tournaments (9 hours)

Domination concepts and other variants:- Dominating sets in graphs, domination number of standard graphs, Minimal dominating set, Bounds of domination number in terms of size, order, degree, diameter, covering and independence number, Domatic number, domatic number of standard graphs. (9 hours)

**TEXT BOOKS**


1. F. Harary: Graph Theory, Addison -Wesley,1969
2. G.Chartrand and Ping Zhang: Introduction to Graph Theory. McGrawHill, International edition (2005)
3. J.A.Bondy and V.S.R.Murthy: Graph Theory with Applications, Macmillan, London, (2004).


**REFERENCE BOOKS**


1. D.B.West, Introduction to Graph Theory,Pearson Education Asia, 2nd Edition, 2002.
2. Charatrand and L. Lesnaik-Foster: Graph and Digraphs, CRC Press (Third Edition), 2010.
3. T.W. Haynes, S.T. Hedetneime and P. J. Slater: Fundamental of domination in graphs, Marcel Dekker. Inc. New York.1998.
4. J. Gross and J. Yellen: Graph Theory and its application, CRC Press LLC, Boca Raton, Florida, 2000.
5. Norman Biggs: Algebraic Graph Theory, Cambridge University Press (2nd Ed.)1996.
6. Godsil and Royle: Algebraic Graph Theory: Springer Verlag, 2002.
7. N. Deo: Graph Theory: Prentice Hall of India Pvt. Ltd. New Delhi – 1990

## DESIGN AND ANALYSIS OF ALGORITHMS
**(4 Hours/week)**

Preliminaries: Introduction to algorithms; Analyzing algorithms: space and time complexity; growth functions; summations; recurrences; sets, asymptotic etc.Basic data structures: Lists, Stacks, Queues, Trees, Heaps and applications. Sorting, searching and selection: Binary search, insertion sort, merge sort, quicksort, Radix sort, counting sort, heap sort, etc. Median finding using quick-select, Median of medians. (12 hours)

Graph algorithms: Depth-first search; Breadthfirst search; Backtracking; Branch-and-bound, etc. (10 hours)

Algorithm design: Divide and Conquer: Greedy Algorithms: some greedy scheduling algorithms, Dijkstra's shortest paths algorithm, Kruskal's minimum spanning tree algorithm. (12 hours)

Dynamic programming: Elements of dynamic programming; The principle of optimality; The knapsack problem; dynamic programming algorithms for optimal polygon triangulation, optimal binary search tree, longest common subsequence, Shortest paths; Chained matrix multiplication, all pairs of shortest paths. (16 hours)

Introduction to NP-Completeness: Polynomial time reductions, verifications, verification algorithms, classes P and NP, NP-hard and NP-complete problems. (10 hours)

### TEXT BOOKS


1. T. Cormen, C. Leiserson, R. Rivest and C. Stein, Introduction to Algorithms, MIT Press, 2001.
2. David Harel, Algorithms, The spirit of Computing, Addison-Wesley, Langman, Singapore, Pvt.Ltd.India, 2000.


### REFERENCE BOOKS


1. Baase S and Gelder, A.V, computer Algorithms, Addition- Wesle Langman Singapore, Ptv. Ltd. India, 2000.
2. Garey, M.R, and Johnson, D.S, Computers and Intractability: A Guide to the Theory of NP-Completeness, W. H. Freeman, San Francisco,1976.
3. R. Sedgewick, Algorithms in C++, Addison- Wesley, 1992.